Algebra 1B

Pennington & Reeves

Apr 27-May 15

10.2 a
Simplifying Radicals

Who uses this?
Forest, conservation and logging workers
Economists
Actuary
Statistician
Engineer
Insurance underwriters

Vocabulary

$$\sqrt{a}$$

radical symbol

radicand
the number underneath the radical

radical expression
An expression that has radicals in it

Ways to Simplify
A Radical Expression

Option 1

- **Find the prime factorization of the radicand**
- **Every 2 equal factors under the radical sign is equivalent to one of those factors outside of the radical**

Examples

$\sqrt{75} = \sqrt{3 \cdot (5 \cdot 5)} = 5\sqrt{3}$

$\sqrt{300} = \sqrt{(2 \cdot 2) \cdot 3 \cdot (5 \cdot 5)} = 2 \cdot 5\sqrt{3} = 10\sqrt{3}$

$\sqrt{60} = \sqrt{(2 \cdot 2) \cdot 3 \cdot 5} = 2\sqrt{15}$

$4\sqrt{2} \cdot \sqrt{2} = 4\sqrt{(2 \cdot 2)} = 4 \cdot 2 = 8$

Ways to Simplify A Radical Expression

Option 2

- **Express the radicand using perfect square factors**
- **Use the Product Property of Square Roots to simplify**

Examples

$\sqrt{75} = \sqrt{25 \cdot 3} = \sqrt{25}\sqrt{3} = 5\sqrt{3}$

$\sqrt{300} = \sqrt{100 \cdot 3} = \sqrt{100}\sqrt{3} = 10\sqrt{3}$

$\sqrt{60} = \sqrt{4 \cdot 15} = \sqrt{4}\sqrt{15} = 2\sqrt{15}$

$4\sqrt{2} \cdot \sqrt{2} = 4\sqrt{2 \cdot 2} = 4\sqrt{4} = 4 \cdot 2 = 8$

Examples

$\sqrt{2} \cdot \sqrt{10} = \sqrt{20} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$

$\sqrt{15} \cdot \sqrt{3} = \sqrt{45} = \sqrt{9 \cdot 5} = 3\sqrt{5}$

$\sqrt{12} \cdot 3\sqrt{6} = 3\sqrt{72} = 3\sqrt{36 \cdot 2} = 18\sqrt{2}$

$7\sqrt{15} \cdot \left(-2\sqrt{21}\right) = -14\sqrt{315} = -14\sqrt{9 \cdot 35} = -42\sqrt{35}$

Examples

$\sqrt{x^3}$

$\sqrt{x \cdot x \cdot x}$

$x^1\sqrt{x}$

$\sqrt{x^2y^4}$

$\sqrt{x \cdot x \cdot y \cdot y \cdot y \cdot y}$

xy^2

$\sqrt{x^5y^9}$

$\sqrt{x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y}$

$x^2\,y^4\sqrt{xy}$

$\sqrt{16x^4y^6}$

$4x^2y^3\sqrt{\ }$

Examples

$$\sqrt{48x^9}$$

$$\sqrt{2\cdot 2\cdot 2\cdot 2\cdot 3\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x}$$

$$4x^4\sqrt{3x}$$

$$3\sqrt{5x}\cdot\sqrt{15x^3}$$

$$3\sqrt{75x^4}$$

$$3\sqrt{5\cdot 5\cdot 3\cdot x\cdot x\cdot x\cdot x}$$

$$15\,x^2\sqrt{3}$$

Examples

$$3\sqrt{12x^2}\cdot 2\sqrt{20x^5}$$

$$6\sqrt{240x^7}$$

$$6\sqrt{2\cdot 2\cdot 2\cdot 2\cdot 5\cdot 3\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x}$$

$$24\,x^3\sqrt{15x}$$

WS 10.2
Reducing Radicals

Name_____
Date _____ **Per** ____

Reduce each answer to simplest radical form.

1) $\sqrt{24}$

2) $4\sqrt{32}$

3) $\sqrt{150}$

4) $\sqrt{700}$

5) $-3\sqrt{125}$

6) $4\sqrt{8}$

7) $5\sqrt{80}$

8) $-6\sqrt{250}$

9) $10\sqrt{27}$

10) $25\sqrt{50}$

11) $15\sqrt{60}$

12) $-5\sqrt{44}$

13) $\sqrt{27}$

14) $\sqrt{75}$

15) $\sqrt{50}$

16) $\sqrt{8}$

17) $\sqrt{80}$

18) $\sqrt{60}$

19) $2\sqrt{45}$

20) $4\sqrt{12}$

10.3 a
Operations with Radicals

Adding & Subtracting Like Radicals

Vocab: like radicals

- **radicals that contain the same radicand**

Examples

$2\sqrt{5}$ and $5\sqrt{5}$

$5\sqrt{x-3}$ and $4\sqrt{x-3}$

Steps

1. **Simplify each radical expression**
2. **Combine like radicals**

- **Note: This is extremely similar to combining like terms**

$5x+4x=9x$ $5x+6y-4x-3y=x+3y$

$5\sqrt{2}+4\sqrt{2}=9\sqrt{2}$

$5\sqrt{2}+6\sqrt{3}-4\sqrt{2}-3\sqrt{3}=\sqrt{2}+3\sqrt{3}$

Example1

$2\sqrt{20}+\sqrt{45}$

$2\sqrt{2\cdot2\cdot5}+\sqrt{3\cdot3\cdot5}$

$2(2)\sqrt{5}+3\sqrt{5}$

$4\sqrt{5}+3\sqrt{5}$

$\boxed{7\sqrt{5}}$

Example2

$$4\sqrt{27}-5\sqrt{12}$$
$$4\sqrt{9}\sqrt{3}-5\sqrt{4}\sqrt{3}$$
$$4(3)\sqrt{3}-5(2)\sqrt{3}$$
$$\underline{12\sqrt{3}}-\underline{10\sqrt{3}}$$
$$\boxed{2\sqrt{3}}$$

Example3

$$7\sqrt{18}+3\sqrt{50}$$
$$7\sqrt{9}\sqrt{2}+3\sqrt{25}\sqrt{2}$$
$$7(3)\sqrt{2}+3(5)\sqrt{2}$$
$$\underline{21\sqrt{2}}+\underline{15\sqrt{2}}$$
$$\boxed{36\sqrt{2}}$$

Teacher: _____

Algebra 1-B
Adding & Subtracting Radicals WS 10.3 a

Name _____

Date _____ Period _____

Add or subtract and simplify.

1. $-9\sqrt{7}-2\sqrt{7}$
2. $3\sqrt{6}+4\sqrt{2}$
3. $-11\sqrt[3]{21}-11\sqrt{21}$
4. $12\sqrt{15}-10\sqrt{15}$
5. $12\sqrt[3]{7}-14\sqrt[3]{7}$
6. $-3\sqrt{17}+4\sqrt{17}$
7. $-5\sqrt{3}+6\sqrt{27}$
8. $-2\sqrt{6}-2\sqrt{24}$
9. $7\sqrt{6}-(-9\sqrt{54})$
10. $-2\sqrt{12}+13\sqrt{3}$
11. $5\sqrt[3]{3}+5$
12. $5\sqrt{5}-\sqrt{125}$
13. $3\sqrt{8}-6\sqrt{2}$
14. $-17\sqrt{1}+17\sqrt{1}$
15. $-35\sqrt{20}-9\sqrt[4]{5}$
16. $2\sqrt{45}-2\sqrt{5}$
17. $3\sqrt{18}+3\sqrt{12}+2\sqrt{27}$
18. $-3\sqrt{5}-\sqrt{6}-\sqrt{5}$

10.3 b
Operations with Radicals
(multiplying and dividing)

Who uses this?
Physicists
Astronomers
Actuaries
Economists

Examples 1-3

$$\sqrt{\frac{36}{49}} = \frac{\sqrt{36}}{\sqrt{49}} = \frac{6}{7}$$

$$\sqrt{\frac{100}{225}} = \frac{\sqrt{100}}{\sqrt{225}} = \frac{10}{15} = \frac{2}{3}$$

$$\sqrt{\frac{72}{40}} = \sqrt{\frac{9}{5}} = \frac{\sqrt{9}}{\sqrt{5}} = \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{\sqrt{25}} = \frac{3\sqrt{5}}{5}$$

Examples 4-6

$$\frac{\sqrt{12}}{\sqrt{15}} = \sqrt{\frac{12}{15}} = \sqrt{\frac{4}{5}} = \frac{\sqrt{4}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{25}} = \frac{2\sqrt{5}}{5}$$

$$\sqrt{\frac{18y}{36y^3}} = \sqrt{\frac{1}{2y^2}} = \frac{\sqrt{1}}{\sqrt{2y^2}} = \frac{1}{y\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2y}$$

$$\frac{\sqrt{24}}{\sqrt{3n}} = \sqrt{\frac{8}{n}} = \frac{\sqrt{8}}{\sqrt{n}} = \frac{2\sqrt{2}}{\sqrt{n}} \cdot \frac{\sqrt{n}}{\sqrt{n}} = \frac{2\sqrt{2n}}{n}$$

Example 7

$$\sqrt{\frac{16a^2}{4b^2}} = \sqrt{\frac{4a^2}{b^2}} = \frac{\sqrt{4a^2}}{\sqrt{b^2}} = \frac{2a}{b}$$

Example 8

$$\sqrt{3}\left(\sqrt{12}+4\right)$$

$$\sqrt{3\cdot 12}+4\sqrt{3}$$

$$\sqrt{2\cdot 2\cdot 3\cdot 3}+4\sqrt{3}$$

$$2\cdot 3+4\sqrt{3}$$

$$\boxed{6+4\sqrt{3}}$$

Example 9

$$\sqrt{8}\left(\sqrt{5}+4\right)$$

$$\sqrt{8\cdot 5}+4\sqrt{8}$$

$$\sqrt{2\cdot 2\cdot 2\cdot 5}+4\sqrt{2\cdot 2\cdot 2}$$

$$\boxed{2\sqrt{10}+8\sqrt{2}}$$

Example 10

$$\left(4+4\sqrt{5}\right)\left(1+\sqrt{5}\right)$$

$$4+4\sqrt{5}+4\sqrt{5}+4\sqrt{5\cdot 5}$$

$$4+8\sqrt{5}+20$$

$$\boxed{24+8\sqrt{5}}$$

Example 11

$$\left(\sqrt{2}-3\right)^2$$

$$\left(\sqrt{2}-3\right)\left(\sqrt{2}-3\right)$$

$$\sqrt{2\cdot 2}-3\sqrt{2}-3\sqrt{2}+9$$

$$2-6\sqrt{2}+9$$

$$\boxed{11-6\sqrt{2}}$$

Example 12

$$\left(\sqrt{2}-3\right)\left(\sqrt{2}+3\right)$$

$$\sqrt{2\cdot 2}+3\sqrt{2}-3\sqrt{2}-9$$

$$2-9$$

$$\boxed{-7}$$

Example 13

$$\frac{12}{\sqrt{11}-\sqrt{7}} \cdot \frac{\sqrt{11}+\sqrt{7}}{\sqrt{11}+\sqrt{7}}$$

$$\frac{12\sqrt{11}+12\sqrt{7}}{\sqrt{11\cdot 11}+\sqrt{11\cdot 7}-\sqrt{7\cdot 11}-\sqrt{7\cdot 7}}$$

$$\frac{12\sqrt{11}+12\sqrt{7}}{11-7} \qquad \frac{12\sqrt{11}+12\sqrt{7}}{4} \qquad \frac{12\sqrt{11}}{4}+\frac{12\sqrt{7}}{4}$$

$$3\sqrt{11}+3\sqrt{7}$$

Example 14

$$\frac{8}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\frac{8\sqrt{3}+8}{\sqrt{3\cdot 3}+\sqrt{3}-\sqrt{3}-1}$$

$$\frac{8\sqrt{3}+8}{3-1} \qquad \frac{8\sqrt{3}+8}{2} \qquad \frac{8\sqrt{3}}{2}+\frac{8}{2}$$

$$4\sqrt{3}+4$$

Teacher: _____

Algebra 1-B
WS 10.3 b Dividing Radicals

Name _____

Date _____ Period ____

Simplify each radical.

1. $\left(5-3\sqrt{33}\right)\left(5+3\sqrt{33}\right)$

2. $\dfrac{\sqrt{5}}{3-\sqrt{8}}$

3. $\left(3\sqrt{11}-2\sqrt{14}\right)\left(3\sqrt{11}+2\sqrt{14}\right)$

4. $\dfrac{3}{4+4\sqrt{5}}$

5. $\dfrac{4}{\sqrt{5}+\sqrt{2}}$

6. $\dfrac{56}{\sqrt{3}+\sqrt{10}}$

7. $\dfrac{15}{\sqrt{11}-\sqrt{6}}$

8. $\dfrac{-20}{\sqrt{6}-\sqrt{2}}$

9. $\dfrac{10}{\sqrt{22}+4}$

10. $\dfrac{36}{4-\sqrt{7}}$

11. $\dfrac{2+\sqrt{5}}{6-\sqrt{7}}$

12. $\dfrac{2\sqrt{3}+3\sqrt{5}}{3\sqrt{3}-5\sqrt{5}}$

13. $\frac{15}{2\sqrt{7}}$

14. $\sqrt{\frac{18}{49}}$

15. $\sqrt{\frac{10x^3}{128x}}$

16. $\frac{40}{\sqrt{5}}$

17. $\sqrt{\frac{1}{80}}$

18. $\frac{3\sqrt{10}}{\sqrt{15}}$

19. $\frac{6\sqrt{11n^2}}{\sqrt{3n}}$

20. $\frac{2}{\sqrt[3]{100y}}$

21. $\frac{\sqrt{5}}{\sqrt{3}}$

22. $\frac{4\sqrt{2}}{3\sqrt{7}}$

23. $\frac{\sqrt{2}-\sqrt{8}}{4\sqrt{9}}$

24. $\frac{3+\sqrt[3]{3}}{\sqrt[3]{9}}$